

# What do we know about neutrinoless double-beta decay nuclear matrix elements?

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The detection of neutrinoless double-beta decay will establish the Majorana nature of neutrinos. In addition, if the nuclear matrix elements of this process are reliably known, the experimental lifetime will provide precious information about the absolute neutrino masses and hierarchy. I review the status of nuclear structure calculations for neutrinoless double-beta decay matrix elements, and discuss some key issues to be addressed in order to meet the demand for accurate nuclear matrix elements.

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# 1 Neutrinoless double-beta decay

Neutrinoless double-beta ( $0\nu\beta\beta$ ) decay is a very special process. Most importantly, the experimental detection of this lepton-number violating decay will proof the Majorana nature of neutrinos, this is, that they are their own antiparticle. In addition, the lifetime of the  $0\nu\beta\beta$  decay is related to the neutrino masses so that its measurement will also probe the unknown absolute neutrino mass and hierarchy.

However, there is yet another ingredient in the connection between the  $0\nu\beta\beta$  decay lifetime and the neutrino mass: since it is a nuclear decay, the lifetime naturally depends on the nuclear matrix element (NME) of the transition. Overall, the  $0\nu\beta\beta$  decay half-life can be written as [1]

$$\left[ T_{1/2}^{0\nu\beta\beta} (0_i^+ \rightarrow 0_f^+) \right]^{-1} = G^{0\nu\beta\beta} |M^{0\nu\beta\beta}|^2 m_{\beta\beta}^2, \quad (1)$$

with  $M^{0\nu\beta\beta}$  the NME,  $m_{\beta\beta}$  a combination of the absolute neutrino masses and the neutrino mixing matrix, and  $G^{0\nu\beta\beta}$  a well-known phase-space factor [2]. It is apparent, therefore, that for  $0\nu\beta\beta$  decay experiments [3, 4, 5, 6] to be able to unveil the neutrino masses, the NMEs of the decay have to be accurately known. Is this presently the case?

To answer this question, let us recall that the NME in the closure approximation is [1]

$$M^{0\nu\beta\beta} = \langle 0_f^+ | \hat{O}^{0\nu\beta\beta} | 0_i^+ \rangle. \quad (2)$$

Therefore, a reliable NME relies on two independent parts: first, the nuclear structure of the transition initial and final states; second, the evaluation of the decay operator  $\hat{O}^{0\nu\beta\beta}$  between these states. In the following, these two parts are analysed separately.

## 2 The initial and final nuclei: nuclear structure

First let us focus on the impact of the nuclear structure of the initial and final states in the  $0\nu\beta\beta$  decay NMEs. Very different nuclear structure approaches have been used to study this process. Figure 1 shows an updated comparison of the main NME calculations obtained with various nuclear structure frameworks [7, 8, 9, 10, 12, 11, 13, 14]. The differences are about a factor of two to three, or three to four units.

Among the smallest NMEs are those from shell model calculations. The nuclear shell model is very successful in describing nuclear masses, low-lying excited states, electromagnetic transitions and single- $\beta$  decays over a wide range of nuclei [15]. These calculations can include very rich nuclear structure correlations. However, they are typically performed in a rather limited configuration space of one major harmonic-oscillator shell, while the remaining orbitals are taken into account only perturbatively.

In order to explore the impact of the size of the configuration space in shell model NMEs [16], a very recent work by the Tokyo group focused on the  $0\nu\beta\beta$  decay of  $^{48}\text{Ca}$  [8].

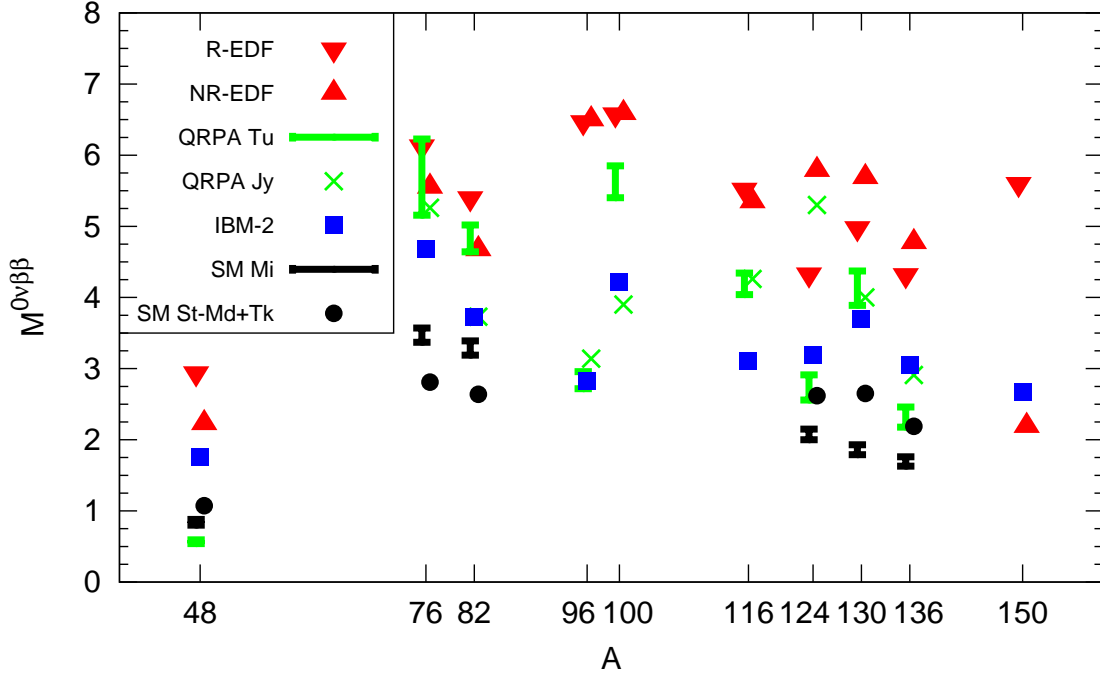


Figure 1: NMEs for the  $0\nu\beta\beta$  decays of  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{96}\text{Zr}$ ,  $^{100}\text{Mo}$ ,  $^{116}\text{Cd}$ ,  $^{124}\text{Sn}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$  and  $^{150}\text{Nd}$ , shown according to their mass number  $A$ . Results are shown for the shell model calculations of the Strasbourg-Madrid [7] and Tokyo [8] groups (SM St-Ma+Tk), and the Michigan group [9] (SM Mi), the interacting boson model [10] (IBM-2), the quasiparticle random-phase approximation approach of the Jyväskylä [11] (QRPA Jy) and Tübingen [12] (QRPA Tu) groups, and the non-relativistic [13] and relativistic [14] energy density functional frameworks (NR-EDF and R-EDF, respectively).

Previous studies used a configuration space comprising the  $pf$ -shell, this is, assuming a  $^{40}\text{Ca}$  core with eight neutrons in the four  $pf$ -shell orbitals [7]. In Ref. [8], the configuration space was expanded to include two major harmonic-oscillator shells, adding the  $sd$ -shell to the  $pf$ -shell. In this case, a core of  $^{16}\text{O}$  was assumed, allowing up to total  $2\hbar\omega$  proton and neutron cross-shell excitations from the  $sd$ - into the  $pf$ -shell. As a result, the size of the diagonalization needed to describe the daughter nucleus  $^{48}\text{Ti}$  increases from less than  $10^6$  to over  $10^9$ , at the limit of present capabilities. The effect of the extended calculation compared the one-major-shell one is illustrated in Fig. 2. The NME increases by about 30%, with the enhancement produced by additional cross-shell pairing correlations incorporated in the enlarged configuration space [8]. However, the improved NME is still far from the results of other approaches, suggesting that the size of the shell model configuration space may not explain the disagreement between NME calculations.

Another important aspect for  $0\nu\beta\beta$  decay NMEs are nuclear structure correlations. Ac-

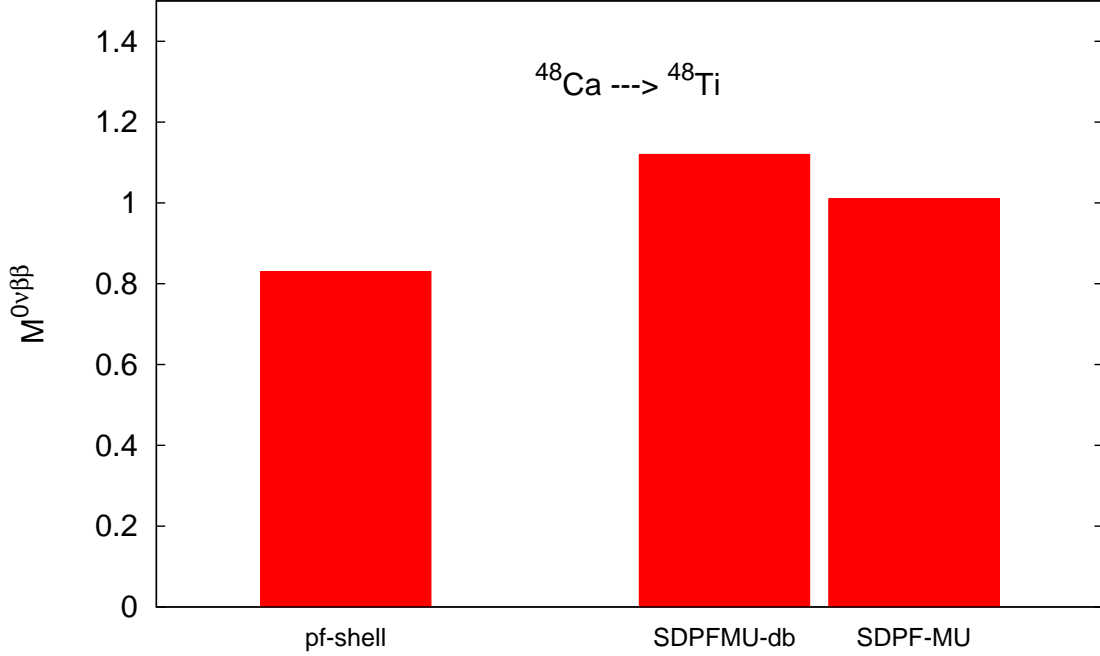


Figure 2: Shell model NME for the  $0\nu\beta\beta$  decay of  $^{48}\text{Ca}$  in one major harmonic-oscillator shell (left), and in two calculations in two major harmonic-oscillator shells (right), from Ref. [8]. The enhancement of the NME in the enlarged configuration space is about 30%.

tually in some cases the disagreement between NME calculations is strongly reduced when they are restricted to uncorrelated (and therefore too simplistic) initial and final states [17]. As already pointed out in the case of the  $^{48}\text{Ca}$  decay, pairing correlations are very important in this process. Proton-proton and neutron-neutron pairing correlations favour  $0\nu\beta\beta$  decay: the more of these correlations in the initial and final states, the larger the NMEs [18, 13]. This explains why the additional pairing correlations captured in the two major-shell calculation enhance the  $^{48}\text{Ca}$   $0\nu\beta\beta$  decay NME. Similarly, if pairing correlations are overestimated, the NMEs will be overpredicted [18].

Proton-neutron pairing correlations (more precisely isoscalar pairing correlations) also impact  $0\nu\beta\beta$  decay [19]. In contrast to like-particle pairing, neglecting isoscalar pairing results in overpredicted NMEs. This may be somewhat surprising because proton-neutron pairing correlations are usually not very relevant in nuclear structure. However, in single- $\beta$  decays and  $\beta\beta$  decays, isoscalar pairing is crucial because neglecting this term breaks the spin-isospin  $\text{SU}(4)$  symmetry of the operators, which are therefore especially sensitive to these correlations. A proper treatment of isoscalar pairing is important for  $0\nu\beta\beta$  decay because energy density functional methods (that predict the largest NMEs as shown in Fig. 1) and the interacting boson model do not include these correlations explicitly. Without a dedicated calculation it is difficult to quantify the impact of isoscalar pairing correlations

in the NMEs, but a recent shell model study suggests that the effect could be as large as a 50% NME reduction [20].

In addition, quadrupole correlations related to deformation are also relevant for  $0\nu\beta\beta$  decay [21, 22]. In this case, quadrupole correlations reduce the NMEs, especially when the deformation of the initial and final states is different. The treatment of deformation may explain the different NMEs between the two energy density functional calculations for  $^{150}\text{Nd}$ , the only strongly deformed  $0\nu\beta\beta$  decay candidate.

All NMEs available so far are based on phenomenological nuclear structure calculations. One of the main advances in low-energy nuclear physics in the recent decade is the capability of performing first principles calculations based on the underlying theory of the strong interaction, QCD, combined with improved many-body methods that make use of state-of-the-art computational resources. For instance, nuclear structure calculations using interactions derived from chiral effective field theory (EFT) [23], an effective theory based on the symmetries of QCD, have been very successful in describing and predicting properties of medium-mass nuclei up to calcium [24]. Moreover, in selected cases the many-body problem can be solved with all nucleons explicitly included. These ab initio approaches are not able to provide  $0\nu\beta\beta$  decay NMEs yet, but they will be able to do so in the near future. As a first step, single- $\beta$  decays of medium-mass nuclei, albeit for isotopes lighter than those relevant for  $0\nu\beta\beta$  decay experiments, are already available [25].

### 3 The transition operator: two-body corrections

The different NMEs discussed in Sec. 2 assume a common transition operator entirely consisting of axial and vector weak one-body (1b) currents. However, studies of light nuclei with mass number  $A \lesssim 10$  manifest the need to go beyond the 1b level to describe magnetic moments and transitions [26], or single- $\beta$  decays [27].

Chiral EFT, in addition to a theory of nuclear interactions, also predicts how nucleons interact with external probes, in particular via the weak interaction. Since chiral EFT is an effective theory, different terms are organized in orders in the expansion coefficient  $Q$ . Chiral EFT predicts that two-body (2b), or meson-exchange currents enter  $0\nu\beta\beta$  decay at order  $Q^2$  in the vector current and at order  $Q^3$  in the axial current [28]. This is important because the 1b terms used in standard  $0\nu\beta\beta$  decay calculations correspond to 1b currents to order  $Q^2$ , and the next 1b current contributions only appear at order  $Q^4$ . Figure 3 schematically shows the diagrams of the leading 1b and 2b currents to order  $Q^3$ .

The 1b terms relevant for  $0\nu\beta\beta$  decay to order  $Q^2$  are [29]

$$\begin{aligned} V_{1b} &= \tau^- g_V(q), & \mathbf{V}_{1b} &= \tau^- \left[ (1 + g_M) \frac{(-i\boldsymbol{\sigma} \times \mathbf{q})}{2M} \right], \\ \mathbf{A}_{1b} &= \tau^- [g_A(q)\boldsymbol{\sigma} - g_P(q)(\mathbf{q} \cdot \boldsymbol{\sigma})\mathbf{q}], \end{aligned} \quad (3)$$

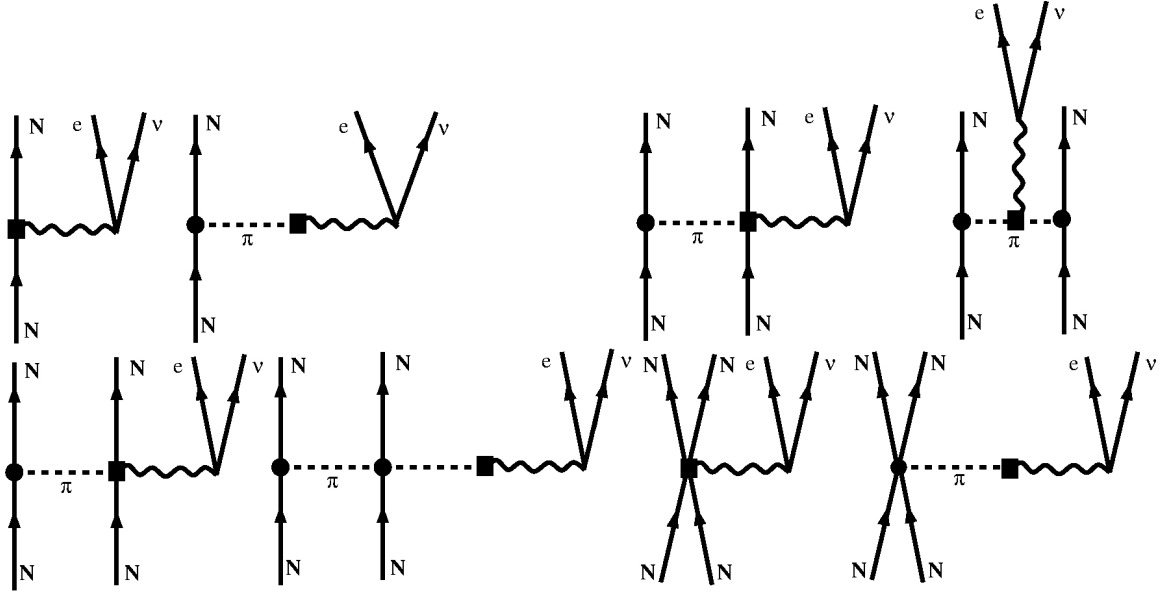


Figure 3: Diagrams corresponding to the 1b currents (upper left part), vector 2b currents (upper right part) and axial 2b currents (lower part) relevant for  $0\nu\beta\beta$  decay.

with  $M$  the nucleon mass and  $\mathbf{q}$  the momentum-transfer of the transition. At vanishing momentum transfer  $g_V(0) = 1$  because of the conserved vector current, and  $g_A(0) = g_A$ , the axial coupling constant. The coefficient  $g_M$  accounts for the isovector anomalous magnetic moment of the nucleon, and  $g_P(q)$  is fixed by the Goldberger-Treiman relation [29].

The leading correction to the 1b terms in Eq. 3 are 2b currents. The evaluation of these 2b terms in  $0\nu\beta\beta$  decay is challenging, because in general they will lead to a four-body operator. As an attempt to estimate the importance of 2b effects in  $0\nu\beta\beta$  decay, the easiest approach is to perform a normal-ordering approximation over a spin-isospin symmetric reference state (Fermi gas) [29], which results in an effective 1b current coming from the 2b terms. The result can be easily compared to the leading 1b contributions:

$$\begin{aligned} \mathbf{V}_{2b}^{\text{NO}} &= \tau^- \left[ \delta m(q) \frac{(-i\boldsymbol{\sigma} \times \mathbf{q})}{2M} \right], \\ \mathbf{A}_{2b}^{\text{NO}} &= \tau^- [\delta a(q)\boldsymbol{\sigma} - \delta p(q)(\mathbf{q} \cdot \boldsymbol{\sigma})\mathbf{q}], \end{aligned} \quad (4)$$

where  $\delta a(q)$ ,  $\delta p(q)$  and  $\delta m(q)$  can be evaluated with the low-energy chiral EFT couplings. Thus, in this approximation the 2b currents amount to a momentum-transfer dependent modification of the magnetic, Gamow-Teller and pseudoscalar 1b terms.

The most important 2b term is the correction to the Gamow-Teller  $\tau^- \boldsymbol{\sigma}$  term. This 2b contribution is enhanced due to the low-lying  $\Delta$ -isobar excitation [29], and its effect is to reduce the strength of the 1b Gamow-Teller term. A similar need to reduce the strength of this operator is well-known in nuclear structure calculations trying to reproduce the experi-

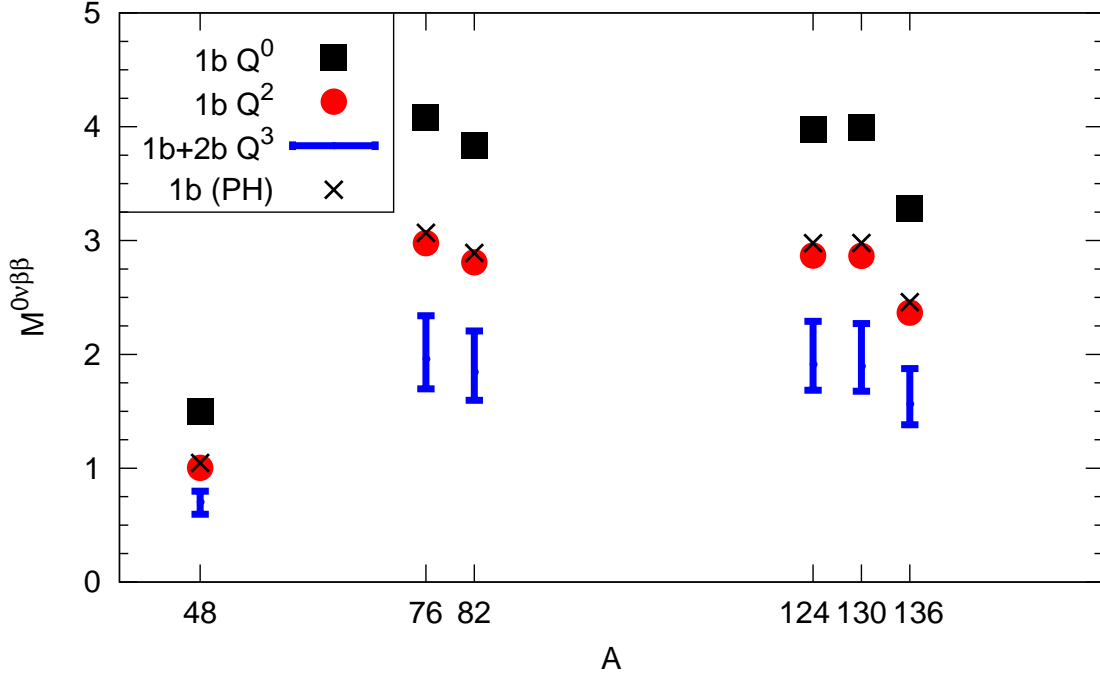


Figure 4: Shell model NMEs for the  $0\nu\beta\beta$  decays of  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{124}\text{Sn}$ ,  $^{130}\text{Te}$  and  $^{136}\text{Xe}$ , shown according to their mass number  $A$ . The different calculations include chiral 1b currents to leading order  $Q^0$  (black squares) and order  $Q^2$  (red circles), and also 2b axial and vector currents (blue bars) which are the only additional contributions to order  $Q^3$ . The NMEs are compared with those obtained with phenomenological 1b currents (black crosses), roughly corresponding to a  $Q^2$  1b current calculation.

mental lifetime of Gamow-Teller transitions, a phenomenon usually referred to as Gamow-Teller quenching [15]. Even though there may be additional mechanisms leading to this quenching, such as corrections due to the limitations in the nuclear structure calculations, the estimation in Ref. [29] suggests that 2b currents are a significant contribution.

The other two terms in Eq. (4) are relevant at high momentum transfers. They also impact  $0\nu\beta\beta$  decay as in this process typically  $q \sim 200$  MeV due to the virtual nature of the neutrinos [16]. The combined effect of these terms is to partially compensate for the reduction produced by the leading 2b contribution [29].

Figure 4 shows NMEs calculated with chiral EFT 1b and 2b currents in the shell model framework. Only the long-range contributions are included, because they are expected to be dominant (due to its relation to the  $\Delta$ -isobar), and also because the associated chiral EFT couplings are less precisely known for the short-range parts. The NMEs are reduced by the 2b corrections in about 35%, with a relatively large error band stemming from the uncertainties in the chiral EFT couplings. Even though the results in Fig. 4 rely on a sim-

ple normal-ordering approximation they highlight that accurate NME calculations should carefully include 2b currents.

## 4 Conclusions

The  $0\nu\beta\beta$  decay, besides establishing the Majorana nature of neutrinos, has the potential shed light on the absolute neutrino mass and hierarchy. For that purpose, it is critical that the associated NMEs are reliably known. The most recent calculations show NME differences of about a factor of two or three, but the main limitations of the calculations, such as enlarging the shell model configuration space, or including isoscalar pairing correlations have been identified and work is in progress to obtain improved NMEs. In addition, *ab initio* studies based on chiral EFT will soon become available. On the other hand, 2b current corrections to the transition operator are usually neglected, but their effect could be sizeable and they should be included in NME calculations.

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